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Solution by **WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.**

The equation to the tangent plane to the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \dots\dots (1)$$

at the point (x', y', z') is

$$\frac{x'x}{a^2} + \frac{y'y}{b^2} + \frac{z'z}{c^2} = 1 \dots\dots (2).$$

Then

$$\frac{1}{f^2} = \frac{x'^2}{a^4} + \frac{y'^2}{b^4} + \frac{z'^2}{c^4} \dots\dots (3),$$

is the required locus, an ellipsoid concentric with (1).

Also solved by **G. B. M. ZERR**, and **ELMER SCHUYLER**.

126. Proposed by **GEORGE R. DEAN, Professor of Mathematics, University of Missouri School of Mines and Metallurgy, Rolla, Mo.**

Through any fixed point O draw two straight lines at right angles. Let one line cut a given circle at Q , the other at R . Find, by Euclidean methods, the locus of the foot of the perpendicular from O upon the chord QR . Give complete analysis and discussion. Solve also by coördinate geometry.

I. Solution (Analytical) by **WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.**

Let r = the radius of the given circle, a = the distance of its center from O , and take the line through O and the center for the axis of x . Then the coördinate axes being rectangular, the equation to the fixed circle is

$$(x-a)^2 + y^2 = r^2 \text{ or } x^2 + y^2 - 2ax + a^2 - r^2 = 0 \dots\dots (1).$$

If $lx + my = 1 \dots\dots (2)$ be the equation to QR , the equation to the lines OQ and OR is

$$x^2 + y^2 - 2ax(lx + my) + (a^2 - r^2)(lx + my)^2 = 0 \dots\dots (3), \text{ or}$$

$$[1 - 2al + (a^2 - r^2)l^2]x^2 + 2[lm(a^2 - r^2) - am]xy + [1 + (a^2 - r^2)m^2]y^2 = 0 \dots\dots (4).$$

The condition that these lines are perpendicular to each other is

$$2 - 2al + (a^2 - r^2)(l^2 + m^2) = 0 \dots\dots (5).$$

The line through O and perpendicular to (2) is $x/y = l/m \dots\dots (6).$

Making (5) homogeneous by aid of (2), we have,

$$[2x^2 - 2ax + (a^2 - r^2)]l^2 + (4xy - 2ay)lm + [2y^2 + (a^2 - r^2)]m^2 = 0 \dots\dots (7).$$

l and m by (6) being proportional to x and y , (7) easily becomes

$$2x^4 - 2ax^3 + (a^2 - r^2)x^2 + 4x^2y^2 - 2axy^2 + 2y^4 + (a^2 - r^2)y^2 = 0, \text{ or}$$

$$2(x^2 + y^2)^2 - [2ax - (a^2 - r^2)](x^2 + y^2) = 0 \dots\dots (8),$$

giving the two circles

$$x^2 + y^2 = 0 \dots\dots (9), \quad x^2 + y^2 - ax + \frac{1}{2}(a^2 - r^2) = 0 \dots\dots (10)$$

for the required loci.

II. Solution by J. W. YOUNG, Fellow and Assistant in Mathematics, Ohio State University, Columbus, O.; and J. SCHEFFER, A. M., Hagerstown, Md.

To solve the problem by coördinate geometry, we may proceed as follows : Let the fixed point be the origin, and let the axis of x pass through the center of the circle. Its equation is

$$x^2 + y^2 + 2gx + c = 0 \dots\dots (1).$$

Let the chord, QR , be $y = mx + a \dots\dots (2)$.

Make (1) homogeneous by means of the relation $(y - mx)/a = 1$. The resulting equation, viz.,

$$x^2 + y^2 + 2gx \frac{(y - mx)}{a} + \frac{c(y - mx)^2}{a^2} = 0 \dots\dots (3),$$

represents the lines through O , and Q and R .

(3) may be written $(a^2 - 2agm + cm^2)x^2 + (a^2 + c)y^2 + 2(ag - cm)xy = 0$.

But the lines OQ and OR are at right angles. The condition for this is

$$(a^2 - 2agm + cm^2) + (a^2 + c) = 0, \text{ or } 2a^2 + cm^2 - 2agm + c = 0 \dots\dots (4).$$

The perpendicular from O and QR is $y = -(1/m)x \dots\dots (5)$.

We must find the locus of the intersection of (2) and (5), under the condition (4). (5) gives $m = -x/y$. (2) gives $a = y - mx = (y^2 + x^2)/y$.

Substitute for m and a in (4), and we have

$$2 \frac{(y^2 + x^2)^2}{y^2} + c \left(\frac{x^2}{y^2} + 1 \right) + 2g \frac{(x^2 + y^2)x}{y^2} = 0, \text{ or } 2(x^2 + y^2) + 2gx + c = 0,$$

the required locus. This is evidently a circle, with center half way between O and the center of the given circle.